Equivalent Beam-Column Method to Estimate In-Plane Critical Loads of Parabolic Fixed Steel Arches

Jiangang Wei¹; Qingxiong Wu²; Baochun Chen³; and Ton-Lo Wang, M.ASCE⁴

Abstract: The objective of this paper is to investigate the characteristics of critical loads for parabolic fixed steel tubular arches. An advanced nonlinearity finite-element program, taking into account the geometric and material dual nonlinearity, is employed. The influence of nonlinearity and initial crookedness on arch critical load is discussed. It is found that the effect of rise-to-span ratio on the critical load of arch is significant. Therefore, a new equivalent beam-column method is proposed for estimating the corresponding in-plane critical loads of arch, in which a buckling factor $K_1$ is employed to consider influence of rise-to-span ratio and a reduction factor $K_2$ to consider the effect of initial crookedness. Pragmatic formulas and tabulated data are provided based on the present different Chinese design codes. It is proved that the presented method is sufficiently accurate to predict the in-plane critical load of parabolic fixed steel arch subjected to compression or to both bending and compression.

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Introduction

Arches are applied widely in bridges and buildings with their superior structural performance in compression. A number of researchers have studied experimentally and theoretically the stability of arches. The instability phenomenon of arches shown in structural geometry and material is nonlinear in major cases (Langhaar et al. 1954). Moreover, it is difficult to obtain analytic solution of critical loads of an arch for its curved shape. Thus, a simplified method should be provided to estimate the critical loads for arches.

The design principles for arches against their in-plane failure are widely used in design codes (American Institute of Steel Construction, Inc. 2005; China Railway Ministry 2005; Japan Road Association 1980), essentially based on the equivalent beam-column method. This method was founded on the classical buckling analysis and summarized by Timoshenko and Gere (1961) and Austin (1971). The classical buckling analysis assumes that the prebuckling behavior is linear and ignores the effects of prebuckling deformations on the buckling. Austin and Ross (1976) studied the effect of geometrical nonlinearity and found that the equivalent length factor was close to the solution when geometrical nonlinearity was ignored.

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The geometrical nonlinear behavior of fixed steel arches was studied (Bradford et al. 2002; Pi et al. 2002). They obtained analytical solution for the in-plane elastic buckling of circular arches subjected to a central concentrated load or a radial load uniformly distributed along the arch. Based on the design equation and the interaction equation which were proposed for pin-end steel arches, the design rule for fixed steel arch was presented by Pi and Trahair (1999) and Pi and Bradford (2004). More recently, a nonlinear theory to investigate the elastic in-plane buckling of shallow tied circular arch was developed (Bradford et al. 2006).

Indeed, a parabolic profile is funicular for a uniformly distributed gravity loading, and so the arch is subjected only to compressive actions. Because of this, parabolic profiles have found widespread use in the design and construction of arches over a long period of time. However, a lack of understanding of the behavior of parabolic fixed steel arches, and in particular of their propensity to buckle in plane, is a major reason for their infrequent application compared with circular arches.

In-plane design criteria for parabolic arches were proposed, which were related to the axial force and bending moment at the quarter point (Yabuki and Kuranishi 1987). But it is practically difficult to estimate critical loads by this method. The equivalent beam-column method was analyzed in which the ultimate load of column or beam-column was calculated by current design codes (Chen and Qin 2006). Although the calculation results by this method varied in the same way with finite-element method (FEM) results, their distinctions were obvious when the geometric character of arches is neglected. More recently, the effect of creep on buckling of concrete parabolic arches was investigated (Wang et al. 2006). Theoretical research and experiment were carried out respectively on the in-plane elastic stability of a shallow parabolic arch with horizontal spring supports (Bradford et al. 2007; Wang et al. 2007).

This paper is to investigate the characteristic of critical loads for parabolic fixed steel tubular arches using an advanced dual-nonlinearity finite-element (FE) program. Then, the equation simplified by the equivalent beam-column method is proposed to
estimate the critical loads and this method is verified by case studies.

Calculating Parameters

The investigated arch with a parabolic profile is shown in Fig. 1(a), where \(L\) is the span, \(f\) is the rise of arch, \(D\) is the diameter of tubular cross section, \(t\) is the thickness of steel tubular arches, \(f/L\) is the rise-to-span ratio, \(E_s\) is Young’s modulus, and \(f_s\) is the yield stress of steel. Slenderness \(\lambda\) is defined as \(L/r_x\), where \(r_x\) = radius of gyration of the cross section \(r_x = \sqrt{I_x/A_x}\); \(I_x\) = moment of inertia; \(A_x\) = area of the cross section. The arch is fixed at the two ends. The loading arrangement is shown in Figs. 1(b–d).

The investigation of Chen and Yang (2006) showed the popular parameters used in completed arch bridges. So, the key parameters in this paper are based on that, i.e., \(\lambda=L/r_x=100–500\) and \(f/L=0.1–0.5\). The stress-strain relationship of material is shown in Fig. 2, where the postbuckling stiffness is taken as \(E_s/100\). The effect of initial crookedness is included in the analysis. The initial crookedness \(e\) is given by

\[
e = \frac{y_0}{y_0/L/4} = \sin \frac{2\pi x}{L}
\]

where \(x\) and \(y_0\) = coordinates of axial line of arch; and \(y_0/L/4\) = maximum value of initial crookedness curve at the quarter point. It can be expressed as \(e = L/1,000\), \(L/2,000\) and so on. The range of the values is between \(L/3,000\) and \(L/1,000\).

Residual stress is another imperfection for steel arch. It will reduce the critical load and its effect was studied by Pi and Traylor (1996). The results showed that the effects of residual stresses are not important for the steel arches and the maximum reduced percent of critical loads is 4%. Therefore, the effect of residual stresses is not considered in this paper.

Verification of Program

A FE program for three-dimensional frame structure considering material and geometric nonlinearity, NL Beam3D (nonlinear analysis program of Beam3D), is adopted to investigate the in-plane strength of parabolic fixed steel arches (Wu et al. 2007). Incremental-iterative nonlinear analysis makes use of the generalized displacement control method.

The first example applied by Austin and Ross (1976) is the elastic buckling of parabolic arches. The parabolic arch fixed at two ends is subjected to vertical distributed load \(q\). \(L\) is the span of rise, \(f\) is the rise of arch, \(E\) is Young’s modulus of elasticity, and \(I\) is the moment of inertia of cross section. Based on the critical load \(q_{cr} = \alpha(EI/L^3)\), the coefficient \(\alpha\) is \(q_{cr}/(EI/L^3)\). Fig. 3 shows the relationship between the coefficient \(\alpha\) calculated by the NL Beam3D program and the rise-to-span ratio. The results given by Austin and Ross (1976) and Timoshenko and Gere (1961) are also shown in Fig. 3. The results obtained by the NL Beam3D program are fairly close to the results of Austin and Ross (1976).

The second numerical example concerns a circular arch with a span of 160 in. (406.4 cm) and rise of 40 in. (101.6 cm). The cross section is square, with \(b=h=1\) in. (2.54 cm). Young’s modulus is 107 psi (6.896 × 106 N/cm²). The load-displacement

Fig. 1. Geometry of arches and loading types

Fig. 2. Stress-strain relationship of steel material

Fig. 3. Comparison of results of factor \(\alpha\) for the parabolic fixed arches

Fig. 4. Comparison of results for in-plane buckling of elastic arch
curves computed by the NL_Beam3D program and the results given by the perturbation method (Li and Shen 2000) are shown in Fig. 4.

When a hinged arch is subjected to a central concentrated load in the vertical direction, the calculated load-deflection curve shows that the buckling type of the hinged arch is the bifurcation buckling. The buckling load obtained by the present model is quite close to the results by Li and Shen (2000). When this arch is fixed at both ends, the buckling becomes the limit-point buckling. The result attained by the present model also agrees well with that by Li and Shen (2000). Based on these two examples, the NL_Beam3D program has been successfully verified and the accuracy of the program has been confirmed.

### Equivalent Beam-Column Method

A number of investigators have studied theoretically the simplified method to estimate the critical loads of arches. A fixed parabolic arch subjected to uniformly distributed load is only in compression because the bending moment is small enough to be ignored, as shown in Fig. 1(b). In this case, the fixed parabolic arch may buckle suddenly in the loading plane in an antisymmetric bifurcation mode, which is similar to the failure mode of a column in compression.

By first-order analysis, as shown in Fig. 1(b), the relationship between load $q$ and axial force at the quarter point of span $N_{1/4}$ is

$$ q = \frac{N_{1/4}^2}{L} \frac{8f/L}{\sqrt{1 + 4f/L^2}} $$

if $N_{1/4}$ is equivalent to the critical load $N_{cr}$ of a column which is given by

$$ N_{cr} = \frac{\pi^2 EI}{(\mu S)^2} $$

where $S$=length of the arch; and $\mu_s$=equivalent length factor. Substituting Eq. (3) into Eq. (2), the critical load $q_{cr}$ of arch is obtained as

$$ q_{cr} = \frac{\pi^2 EI}{(\mu_s S)^2 L} \times \frac{8f/L}{\sqrt{1 + 4f/L^2}} $$

In practice, $\mu_s$ is used to calculate the critical load $q_{cr}$. As shown in Table 1, for parabolic arches, $\mu_s$ was derived by Austin (1971) based on the investigation of Timoshenko and Gere (1961), in which the linear assumption was adopted. The effect of geometrical nonlinearity was considered in the calculation of arch critical load and those factors have been modified by Austin and Ross (1976).

As can be seen from Table 1, the equivalent length factor $\mu_s$, derived under linear assumption, is identical to the results of which the effect of geometrical nonlinearity was taken into account. It is probably equal to 0.7. This means that a fixed parabolic arch in compression can be equivalent to a column in straight compression with length $L=\mu_s \times S$. In this paper, the arch is equivalent to the corresponding column or the beam-column with length of $\mu_s \times S$ under varied loads.

However, there are some disadvantages in this equivalent beam-column method by applying Eqs. (3) and (4). The effect of initial crookedness on critical load was not taken into account in Eq. (3). This method is only suitable for the arches of which the buckling mode was governed by geometrical nonlinearity, but not suitable for those arches governed by dual nonlinearity. To solve those problems, a dual-nonlinear FEM analyses should be carried out and Eq. (3) should be modified for various cases.

In this paper, an interaction equation, which is widely used in many design codes, is adopted to estimate the ultimate strength of the corresponding beam-column, given by

$$ \frac{N_{1st}}{\varphi_s N_0} + \frac{M_{1st}}{(1 - N_{1st}/N_E)M_0} = 1 $$

where $\varphi_s$=buckling factor; $N_{1st}$ and $M_{1st}$=axial force and moment at the quarter point of arch span obtained by first-order analysis, respectively; $N_E$=Euler force obtained by Eq. (3); and $N_0$ and $M_0$=ultimate strengths of beam-column which can be obtained by design codes.

In Eq. (5), the buckling factor $\varphi_s$ is proposed in design codes based on the summary of ultimate strength of the beam-column structure, which is unreasonable for estimating the critical load of steel arches. Accordingly, the equivalent beam-column method presented in this paper is to propose the values of $\varphi_s$ by FE analysis of steel arches considering dual nonlinearity.

In this paper, $\varphi_s$ is defined as the buckling factor of arch that is indicated as $K_1$ for perfect arch or $K_1 \times K_s$ for arch with initial crookedness. It should be pointed out that the buckling factor $\varphi_s$ in Eq. (5) is derived by the FE analysis of columns when $M=0$. Thus, the buckling factor of arch should also be derived from the analysis of an arch in compression.

### Steel Arches in Compression

#### Critical Loads of Arches

A fixed parabolic arch under a uniformly distributed load is subjected only to compression. The results of critical loads by FE analysis considering dual nonlinearity are shown in Fig. 5, in which the ordinate is the ratio of inelastic results to elastic results.
It can be seen that the inelastic results are almost identical to the elastic ones when the value of slenderness is greater, but the inelastic results are less than the elastic ones when the value of slenderness is smaller. This specific slenderness is defined as critical slenderness.

It means that when the slenderness of the arch is more than the critical slenderness, the elastic buckling appears and the effect of inelastic is minor. But when the slenderness of the arch is less than the critical slenderness, the steel will reach the yield stress before arch buckling, which will cause the descending of critical loads compared with the elastic results. In this case, the effect of geometrical nonlinearity and material nonlinearity should be taken into account to obtain the accurate critical loads of those arches.

Moreover, as shown in Fig. 5, the curves of inelastic critical loads vary with the rise-to-span ratio and the critical slenderness is different depending on the rise-to-span ratio of arch. For this reason, the rise-to-span ratio is a very important parameter for arch in the simplified calculating method.

**Buckling Factor** $K_1$ **for Perfect Arch**

If the arch is in compression with $M=0$ (i.e., the arch is equivalent to a corresponding column), Eq. (5) can be deduced to

$$N_{cr} = K_1 A_s f_y$$

Substituting Eq. (6) into Eq. (2) yields

$$K_1 = \frac{N_{1/4}}{A_s f_y} = \frac{q_{cr} L \sqrt{1 + 4(f/L)^2}}{8f/L}$$

Hence, the buckling factor $K_1$ of arch can be obtained by $q_{cr}$. In Fig. 6, the coarse line represents the results obtained by the classic Euler analysis, and the abscissa $\tilde{\lambda}$ is the slenderness of the corresponding column given by

$$\tilde{\lambda} = \frac{1}{\pi} \frac{\mu r_a S_f}{E_S}$$

where the equivalent length factor $\mu_{r_a}$, derived by Austin (1971) in Table 1, is adopted.

From Fig. 6, $K_1$ matches the results of the classic Euler analysis when the slenderness of the corresponding steel arch is larger than the critical slenderness (defined as $\tilde{\lambda}_p$). But when the slenderness is less than $\tilde{\lambda}_p$, the $K_1$ curve is different from the Euler curve. It indicates that $\tilde{\lambda}$ and $\tilde{\lambda}_p$ are two essential parameters to describe $K_1$ curves. Further, because $K_1$ curves are varied based on the different rise-to-span ratios of the arch, the rise-to-span ratio should be included in calculating the buckling factor $K_1$.

According to the regression of $K_1$ obtained by FE analysis, the $K_1$ curves shown in Fig. 6 can be described as follows:

$$0.215 \leq \tilde{\lambda} < \tilde{\lambda}_p; \quad K_1 = \frac{1}{\tilde{\lambda}_p^2} + C_1 \cdot \left( \frac{\tilde{\lambda} - \tilde{\lambda}_p}{\tilde{\lambda}_p} \right)^2 + C_2 \cdot \left( \frac{\tilde{\lambda} - \tilde{\lambda}_p}{\tilde{\lambda}_p} \right)^2$$

(9a)

$$\tilde{\lambda} \geq \tilde{\lambda}_p; \quad K_1 = \frac{1}{\lambda^2}$$

(9b)

where

$$C_1 = -0.524 + 2.416(f/L) - 2.773(f/L)^2$$

and

$$C_2 = -0.557 + 3.637(f/L) - 4.555(f/L)^2$$

The values of $K_1$ with common rise-to-span ratio and slenderness are shown in Table 2 and they are convenient to be applied in practice.
Reduction Factor $K_2$ Considering Initial Crookedness

If an antisymmetrical initial crookedness, shown in Eq. (1), is applied to the axial line of perfect arch, the failure mode of the arch will be changed from the bifurcation to the limit point (Pi and Trahair 1999; Pi et al. 2002). The maximum load in the load-displacement diagram is taken as the critical load of arches.

The inelastic results of critical load by FEM for varied slenderness of fixed arches ($f/L=0.2$) are shown in Fig. 7, in which the effect of initial crookedness is included. When the initial crookedness is taken into account in the calculation of critical load, the critical load decreases with the slenderness, and there is no critical slenderness in those curves.

The variation of $q_{imp}/q_{per}$ with the slenderness of arch is shown in Fig. 8, where $q_{imp}$ is the critical load of an imperfect arch and $q_{per}$ is that of a perfect arch. It can be seen that the critical loads descend the most when the slenderness reaches the critical slenderness. In this case, the arch is sensitive to initial crookedness mostly because of the $P$-$\Delta$ effect. When the slenderness is larger or smaller than the critical slenderness, the descending of the critical loads caused by initial crookedness is reduced. In these cases, the effect of inelastic or large deflection on carrying capacity of the arch is dominant in the loading procedure.

The buckling factor $K_1$ of perfect arch has been proposed in the previous section. The initial crookedness will cause the descending of critical load for an imperfect arch compared with a perfect one. Consequently, a factor which is defined as $K_2$ (less than or equal to 1) should be adopted to reduce the buckling factor $K_1$. This effect is described in Fig. 8.

Arches Subjected to Bending and Compression

Arches Subjected to Central Load

Typical in-plane inelastic buckling and postbuckling responses of a fixed parabolic steel tubular arch subjected to a central load $P$ without imperfection are shown in Fig. 11. It can be seen that the symmetric limit-point buckling occurs first, followed by the antisymmetric bifurcation buckling on the descending branch of the load-deflection curve. In this paper, the limit-point buckling load is adopted to estimate the critical load of arch.

When the initial antisymmetric crookedness is applied on the arch, only the limit-point buckling occurs and the bifurcation point disappears. The critical loads of perfect or imperfect arches with varied slenderness and rise-to-span ratio are shown in
Fig. 12, in which the ordinate is the ratio of inelastic critical load $P_{\text{inel}}$ to elastic critical load $P_{\text{ela}}$. It can be seen that the loads of dual nonlinearity included are far less than the elastic results, and so the effect of inelastic should be considered in order to obtain the accurate critical load of arch.

Moreover, the effect of initial crookedness on critical load is relatively indistinctive for arches subjected to central load compared with those subjected to vertical uniform load. This is due to the fact that the moment created by the central load is larger than the additional moment created by initial crookedness, and thus the effect of initial crookedness is decreased.

In this paper, the interaction equation for estimating the ultimate loads of steel beam-column recommended in China Structure Ministry /H20849 2003/H20850 is used, shown as

$$
\frac{N}{\varphi_{f} A_{x}} + \frac{\beta_{\text{act}} M}{\gamma_{s} W_{1x} \left(1 - 0.8 \frac{N}{N_{\text{ex}} f_{y}}\right) f_{y}} = 1
$$

(11)

where the buckling factor $\varphi_{f}$ can be obtained from Eq. (9) for the perfect arch or from Eqs. (9) and (10) for the imperfect arch. In Eq. (11), the relationship between axial force $N$ and moment $M$ at the quarter point of arch span can be derived by first-order analysis, and $\gamma_{s}$ is the plastic coefficient of section. $W_{1x}$ is the module of section for edge fiber in compression. $\beta_{\text{act}}$ is the equivalent moment coefficient. $N_{\text{ex}} = \pi^{2}EI/(1.1\lambda)^{2}$. All the parameter definitions can be found in China Structure Ministry (2003).

The parabolic fixed arch subjected to central load $P$ and axial force $N$ as well as moment $M$ at the quarter span of arch can be given as

$$
N_{1/4}^{P} = \cos \theta \cdot \frac{P}{64} \left(64 f L + 15 \frac{L}{f}\right)
$$

(12)

**Table 3.** Reduction Factor $K_{2}$ Considering Initial Crookedness

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<th>$f/L$</th>
<th>0.15</th>
<th>0.2</th>
<th>0.3</th>
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Fig. 10. $K_{1}$ for perfect arch and $K_{1} \times K_{2}$ for imperfect arches
The parabolic fixed arch subjected to half-span load $p$, full-span distributed load $q$, and axial force $N$ as well as moment $M$ at the quarter span of arch can be obtained by the first-order analysis as

$$M_{1/4}^{p,q} = \frac{1}{128}pL^2$$

$$N_{1/4}^{p,q} = qL\left[\frac{1}{\cos \theta} \cdot \frac{1}{8}\frac{f}{L} + \cos \theta \cdot \frac{1}{16}\left(\frac{L}{f} + \frac{5}{L}f\right)\cdot \frac{p}{q}\right]$$

Therefore, the eccentricity $e_0$ at the quarter span of arch is

$$e_0 = \left|\frac{M}{N}\right| = \frac{1}{\cos \theta} \left(\frac{L}{f} + \frac{5}{L}f\right)$$

Substituting Eq. (14) into Eq. (11) yields

$$0.88\bar{\lambda}^2 \left(\frac{N}{\bar{\gamma}_n A_s}\right)^2 = \left(1 + 0.88\bar{\lambda}^2 + \frac{2}{\gamma_s} \cdot \frac{p_0 \cdot D/2}{\sqrt{r_s}}\right) \cdot \frac{N}{\bar{\gamma}_n A_s} + 1 = 0$$

where $\bar{\lambda}$ can be obtained from Eq. (8).

Therefore, the ultimate axial force $N$ of the corresponding beam-column can be calculated by solving Eq. (15), and then the critical load $P$ of arch is obtained by substituting $N$ into Eq. (12).

**Arches Subjected to Half- and Full-Span Uniformly Distributed Load**

Another typical loading is that the arch is in combined bending and axial compression based on the half- and full-span uniformly distributed load, as shown in Fig. 1(d). In this case, the arch will buckle in a limit-point mode. With the increasing ratio of half-span load $p$ to span distributed load $q$, the moment of arch caused by antisymmetric load $p$ is increased, but the axial force is reduced simultaneously. The critical load will descend simultaneously.

The descending of critical load of arch was studied by Deutsch et al. in Galambos’ book (1998). $H_{\alpha,0}$ is used to describe the descending of critical load of arch, in which $H_{\alpha}$ is the critical horizontal reaction and $H_{\alpha,0}$ is the critical horizontal reaction in the case of $p=0$.

It was shown that $\phi$ was only related to $p/q$ instead of $f/L$ and $\lambda$. Although the arches with large slenderness ($L/r_s=1560$) were adopted in their investigation, only the elastic actions were analyzed. For real arch bridges, the slenderness of arch is generally less than 500 (Komatsu and Shinke 1977). As a result, the stress may reach the yield strength before the arch buckling and the effect of inelastic analysis should be taken into account in the analysis.

The values of $\phi$ are shown in Fig. 13(a) with the varied $p/q$ and slenderness by elastic and inelastic analyses. The factor $\phi$ is independent of slenderness in the elastic analysis; however it will decrease when $p/q$ increases in the inelastic analysis. The variation of $\phi$ with varied $f/L$ is shown in Fig. 13(b) obtained from FE analysis. It is also related to $f/L$. Therefore, in the simplified method for estimating critical load of arches under half- and full-span uniformly distributed loads, two parameters, $\lambda$ and $f/L$, should be included.

The initial crookedness of $L/3,000$ applied to arches will cause the descending of critical loads in this type of loading, which is less than 5%, as shown in Fig. 14. In order to estimate the critical load of arches under half- and full-span uniformly distributed loads, the interaction equation of Eq. (11) is also used.

$$M_{1/4}^{p,q} = -\frac{5}{256}PL$$

where $1/\cos \theta = \sqrt{1+(f/L)^2}$. Therefore, the eccentricity $e_0$ at the quarter span of arch is

$$e_0 = \left|\frac{M}{N}\right| = \frac{1}{\cos \theta} \left(\frac{5}{4}\frac{L}{f} + \frac{L}{f}\right)$$

Fig. 13. Effect of $p/q$ on critical loads of arch subjected to half- and full-span distributed loads

Fig. 14. Effect of initial crookedness to critical load of arch subjected to half- and full-span distributed loads
Therefore, by substituting Eq. (18) into Eq. (11), the ultimate axial force $N$ of the corresponding beam-column can be obtained, and then the critical load $q$ of arch is calculated by substituting $N$ into Eq. (16).

**Accuracy of Proposed Method**

The calculation procedure of the presented method is shown in Table 4. More than 300 cases were studied to verify the accuracy of the presented method. The comparisons between the presented and FE methods are shown in Figs. 15–18, which are subjected to three types of loading. The average value and variance of accuracy are shown in Table 5. It indicates that the error is less than 10% and the proposed method provides an accurate and reliable result.

### Table 4. Calculation Procedure of the Presented Method

<table>
<thead>
<tr>
<th>Load types</th>
<th>Uniformly distributed load</th>
<th>Central concentrated load</th>
<th>Half- and full-span uniformly loaded</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>$L = \mu_s S$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Eq. (8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_1, K_2$</td>
<td>Obtained by Eqs. (9) and (10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_{1/4}$</td>
<td>Eqs. (6)</td>
<td>Eqs. (11) and (14)</td>
<td>Eqs. (11) and (18)</td>
</tr>
<tr>
<td>$q_{cr}$ or $p_{cr}$</td>
<td>Eq. (7)</td>
<td>Eq. (12)</td>
<td>Eq. (17)</td>
</tr>
</tbody>
</table>

$$e_0 = \frac{M}{N} = \frac{L \cdot p}{128 \cdot q} \left[ \frac{1}{\cos \theta} \cdot \frac{1}{8} \cdot \frac{1}{L} + \cos \theta \cdot \frac{1}{16} \left( \frac{L}{f} + \frac{5}{2} \cdot \frac{L}{f} \right) \cdot \frac{p}{q} \right]$$  \hspace{1cm} (18)

**Conclusions**

An equivalent beam-column method is proposed for estimating the in-plane critical loads of parabolic fixed arches with steel tubular section. The equations for buckling factor $K_1$ and reduction factor $K_2$ are presented for steel tubular arch. The critical load of arch is related to the rise-to-span ratio of arch, which is taken into account in the buckling factor of arch. An interaction...
The accuracy of the presented method is studied by analyzing the arches subjected to a central concentrated load or half- and full-span uniformly distributed loads. Compared with FE analyses considering axial force and bending moment is proposed for the ultimate load-carrying capacity of concrete filled steel tube (single) arch under in-plane loads. 

Note: “per” and “imp” represent perfect and imperfect arches, respectively.

**References**


